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Optimization for a heat exchanger couple based on the minimum thermal resistance principle

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ABSTRACT

Following the brief introduction to the concept of a physical quantity, entransy, the equivalent thermal resistance of a heat exchanger couple is defined based on the entransy dissipation. The minimum thermal resistance principle is applied to obtain the optimal heat capacity rate of the medium fluid and the optimal allocation of heat exchangers thermal conductance, which correspond to the maximum heat transfer rate in the heat exchanger couple. In addition, analytical expression for the optimal heat capacity rate of the medium fluid is derived, whose reciprocal equals the sum of the reciprocal of the individual heat capacity rate of the hot and cold fluids, just like the case of two electrical capacitors in series. Numerical results in the variation of the thermal resistance and the heat transfer rate with the medium fluid heat capacity rate or the thermal conductance allocation agree with the theoretical analyses. Finally, for comparison, the entropy generation rate is also calculated to obtain its relation with the thermal performance of the heat exchanger couple. The results show that there is no one-to-one correspondence of the minimum entropy generation rate and the maximum heat transfer rate. This indicates that the minimum entropy generation principle cannot be used for optimizing the heat exchanger couple.

1. Introduction

In some particular energy engineering applications, e.g. room heating and air conditioning [1,2], ground-coupled heat pump [3] and primary loop in heavy water reactor of nuclear power plant [4], the energy source and load are at different places or the source and load must be separated in two devices for safety. In these situations, a medium (or hydronic) fluid is required to transfer thermal energy. Fig. 1 is the sketch of such system, which includes heat exchangers at source and load, hydronic fluid, fluid pump and sometimes an expansion chamber.

When the system, as shown in Fig. 1, is used for air heat recovery [5,6], where the supply and exhaust air ducts are at different parts of the room/building, it is usually referred as coil energy recovery loop system or runaround heat exchanger system. Balen et al. [7] set up both experimental rigs and simulation model of a coil energy recovery loop system. The model was validated with the measured data and used for parametric analysis, during which the author found that when the supply and exhaust air have same heat capacity rate, the optimal medium fluid heat capacity rate, which maximizes the heat transfer rate of the system, is the same as that of the air. This result is accordant with the study results of DeJong et al. [8]. Zhou et al. [9] studied the ground-coupled liquid loop heat recovery ventilation system with a simulation model. They found there are optimal medium fluid flow rate and heat transfer area allocation between heat exchangers. Fan et al. [10] studied the run-around heat recovery system using cross-flow flat-plate heat exchangers with aqueous ethylene glycol as the coupling fluid. Through their parametric study with simulation model, they found the system has the highest efficiency when the heat capacity ratio of air and medium fluid are in the range of 0.8–1.2 for different system parameters. However, these authors neither discuss why there exists an optimal heat capacity rate of the medium fluid nor give its analytical expression which is helpful for system optimization design.

In this paper, efforts are focused on "heat exchanger couple" – a simple form of the system in Fig. 1 with one source heat exchanger and one load heat exchanger which are connected by a medium fluid. Based on the concept of entransy and its dissipation introduced by Guo et al. [11], the equivalent thermal resistance of the heat exchanger couple is defined. The minimum thermal resistance principle is applied for optimizing heat transfer performance of the heat exchanger couple. Both analytical and numerical analyses are carried out to optimize the medium fluid heat capacity rate and thermal conductance allocation of the heat exchanger couple. Considering that the minimum entropy generation principle is the commonly used method for heat transfer optimization, the difference between the minimum thermal resistance principle and the minimum entropy generation principle is also discussed in this paper.

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Nomenclature

A	heat transfer area (m^2)	c_1	low temperature fluid at heat exchanger inlet
C	heat capacity rate (W/K)	c_2	low temperature fluid at heat exchanger outlet
c	specific heat (J/kg K)	h	high temperature fluid/heat exchanger with high temperature fluid
E_h	entransy (J K)	h_1	high temperature fluid at heat exchanger inlet
$\dot{E}_h$	entransy flow rate (W K)	h_2	high temperature fluid at heat exchanger outlet
$\dot{E}_{h\phi}$	entransy dissipation rate (W K)	in	flow into an object
$\dot{m}$	mass flow rate (kg/s)	HX	heat exchanger
$\vec{q}$	heat transfer flux vector (W/m^2)	HXs	heat exchanger couple
$\dot{Q}$	heat transfer rate (W)	m	medium fluid
Q_{vh}	thermal energy (J)	m_1	medium fluid at heat exchanger (with high temperature fluid) outlet
R_h	thermal resistance (K/W)	m_2	medium fluid at heat exchanger (with low temperature fluid) outlet
K	overall heat transfer coefficient (W/m^2 K)	opt	optimal value
$\dot{S}_g$	entropy generation rate (W/K)	out	flow out of an object/parameter at outlet
T	temperature (K)		
ΔT_m	log mean temperature difference (K)		
U	internal energy of a object (J)		
Subscripts			
c	low temperature fluid/heat exchanger with low temperature fluid		

2. Optimization of heat transfer process and single heat exchanger

2.1. Entransy and entransy dissipation during heat transfer

In the existing heat transfer textbooks and literatures, there are several quantities to describe heat transfer rate, but there is no concept of efficiency for heat transfer process because the input (for example, high conductivity materials or increased fluid velocity) has different units than the output (increased heat transfer rate or reduced temperature difference). As a result, a heat transfer process can be enhanced, but there is no way to know how to optimize it. In order to overcome this problem, Guo et al. [11] introduced a new physical quantity "entransy", E_h, in terms of the analogy between heat and electrical conduction,

$$E_h = \frac{1}{2} Q_{vh} T = \frac{1}{2} UT \tag{1}$$

where Q_{vh} is the thermal energy which equals to the internal energy U of an object and T is the thermodynamic temperature. Entransy has the meaning of "thermal potential energy" and it describes the heat transfer ability of an object.

In a heat transfer process, together with heat transportation, there is entransy transportation. Consider the steady-state heat conduction without any internal heat source, the energy conservation equation is

$$\nabla \cdot \vec{q} = 0 \tag{2}$$

Multiplying both sides of Eq. (2) with temperature, T, gives

$$\nabla \cdot \left(\vec{q}T\right) - \vec{q} \cdot \nabla T = 0 \tag{3}$$

Eq. (3) is the entransy balance equation with the first term on the left side being the entransy transfer rate associated with the heat transfer while the second term being the local entransy dissipation rate. It can be seen that heat is conserved during heat transfer, but entransy is not conserved and is dissipated due to thermal resistance, which is an alternative irreversibility measurement of heat transfer process not involving the heat-to-work conversion.

2.2. Thermal resistance of heat transfer process

The thermal resistance is commonly defined as temperature difference divided by heat transfer rate. It is widely used for heat transfer analyses for its illustrative characteristic [12,13]. But such thermal resistance concept holds only for one-dimensional, steady-state heat transfer problems without any internal heat source. During electricity transportation, electric energy is dissipated due to electric resistance, which equals to electric energy dissipation divided by electric current squared. Similarly, Guo et al. [11] consider that entransy, which stands for heat transfer ability, is dissipated due to thermal resistance, and the thermal resistance can be expressed as entransy dissipation divided by heat transfer rate squared,

$$R_h = \frac{\dot{E}_{h,in} - \dot{E}_{h,out}}{\dot{Q}^2} = \frac{\dot{E}_{h\phi}}{\dot{Q}^2} \tag{4a}$$

where $\dot{E}_{h,in}$ and $\dot{E}_{h,out}$ are the entransy flow rates into and out of an object, $\dot{E}_{h\phi}$ is the entransy dissipation rate, $\dot{Q}$ is the heat transfer rate. For one-dimensional, steady-state conduction without internal heat source, $\dot{E}_{h\phi} = \Delta T \cdot \dot{Q}$, Eq. (4a) degrades to the traditional expression of thermal resistance as

$$R_h = \frac{\Delta T}{\dot{Q}} \tag{4b}$$

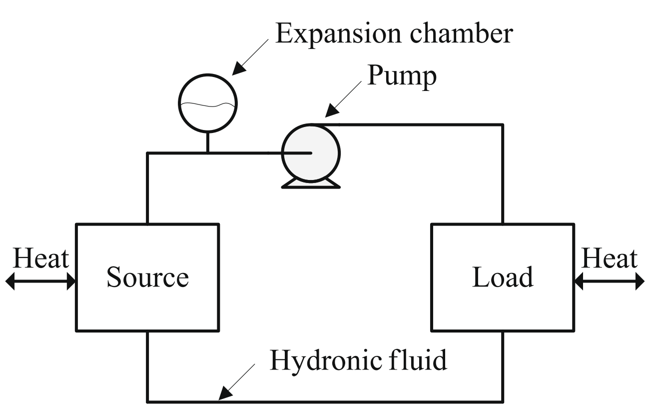

Fig. 1. Schematic of a heat exchanger couple for thermal energy utilization.

So the traditional thermal resistance is a special case of the entransy-dissipation-based thermal resistance which can be applied for multi-dimensional heat transfer problems with/without internal heat sources.

Based on the concept of thermal resistance and using the variational approach, Guo et al. [11] established the minimum thermal resistance principle for heat transfer process. It can be expressed as: for a given set of constraints, the heat transfer process is optimized (i.e. for fixed temperature difference, the heat transfer rate is maximized; for prescribed heat transfer rate, the temperature difference is minimized) when the thermal resistance is minimized. Examples are also given for optimizing the heat conduction and heat convection problems [11].

2.3. Equivalent thermal resistance of a single heat exchanger

In order to discuss and compare the thermal performance among different types of heat exchangers, Liu et al. [14] analyzed the entransy dissipation rate of a single heat exchanger, which is expressed as follows:

$$\dot{E}_{h\phi,HX} = \left(\dot{E}_{h,h1} - \dot{E}_{h,h2}\right) + \left(\dot{E}_{h,c1} - \dot{E}_{h,c2}\right) = \left[\frac{1}{2}\dot{m}_h c_h T_{h1}^2 - \frac{1}{2}\dot{m}_h c_h T_{h2}^2\right] + \left[\frac{1}{2}\dot{m}_c c_c T_{c1}^2 - \frac{1}{2}\dot{m}_c c_c T_{c2}^2\right] \quad (5)$$

where $\dot{E}_{h,h1}, \dot{E}_{h,h2}$ are the inlet and outlet entransy flow rates of high-temperature fluid, $\dot{E}_{h,c1}$, $\dot{E}_{h,c2}$ are the inlet and outlet entransy flow rates of low-temperature fluid. The entransy dissipation rate of the two fluids flowing in the heat exchanger equals to the difference between total entransy entering and leaving the heat exchanger.

With the entransy dissipation expression equation (5), the equivalent thermal resistance of single heat exchanger can be defined as

$$R_{h,HX} = \frac{\dot{E}_{h\phi,HX}}{\dot{Q}_{HX}^2} = \frac{\left[\frac{1}{2}\dot{m}_h c_h T_{h1}^2 - \frac{1}{2}\dot{m}_h c_h T_{h2}^2\right] + \left[\frac{1}{2}\dot{m}_c c_c T_{c1}^2 - \frac{1}{2}\dot{m}_c c_c T_{c2}^2\right]}{\dot{Q}_{HX}^2} \quad (6)$$

Note that this expression for the equivalent thermal resistance is independent of the heat exchanger geometry. Liu et al. [14] also established the connections among the heat exchanger thermal resistance, the fluid heat capacity rate and the heat exchanger effectiveness, which are useful for comparing the thermal performance of different types of heat exchangers.

3. Optimization of a heat exchanger couple

As mentioned, in practical energy engineering applications, thermal energy sources and loads are usually at different places. In these situations, thermal energy is delivered or transported by a hydronic fluid from the sources to loads, which involves heat transfer between several heat exchangers. Heat exchanger couple is the basic formation, as shown in Fig. 2. In heat exchanger A, thermal energy is transferred from the high temperature fluid to the medium fluid and in heat exchanger B the medium fluid conveys energy to the low temperature fluid.

3.1. Thermal resistance of heat exchanger couple

The entransy dissipation rate and the equivalent thermal resistance of heat exchanger A and heat exchanger B in Fig. 2 can be obtained by Eqs. (5) and (6):

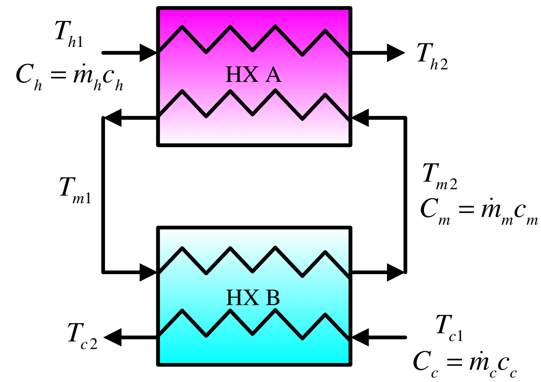

Fig. 2. Heat exchanger couple connected by a hydronic fluid.

$$\dot{E}_{h\phi,h} = \frac{1}{2}C_h T_{h1}^2 - \frac{1}{2}C_h T_{h2}^2 + \frac{1}{2}C_m T_{m2}^2 - \frac{1}{2}C_m T_{m1}^2 \quad (7)$$

$$R_{h,h} = \frac{\dot{E}_{h\phi,h}}{\dot{Q}_h^2} \quad (8)$$

$$\dot{E}_{h\phi,c} = \frac{1}{2}C_c T_{c1}^2 - \frac{1}{2}C_c T_{c2}^2 + \frac{1}{2}C_m T_{m1}^2 - \frac{1}{2}C_m T_{m2}^2 \quad (9)$$

$$R_{h,c} = \frac{\dot{E}_{h\phi,c}}{\dot{Q}_c^2} \quad (10)$$

For ideal steady state, the relations between heat transfer rates/energy changes are:

$$\dot{Q}_{HXs} = \dot{Q}_h = \dot{Q}_m = \dot{Q}_c \quad (11a)$$

$$\dot{Q}_h = C_h(T_{h1} - T_{h2}) \quad (11b)$$

$$\dot{Q}_m = C_m(T_{m1} - T_{m2}) \quad (11c)$$

$$\dot{Q}_c = C_c(T_{h2} - T_{h1}) \quad (11d)$$

The overall entransy dissipation and the equivalent thermal resistance of heat exchanger couple equals

$$\dot{E}_{h\phi,HXs} = \dot{E}_{h\phi,h} + \dot{E}_{h\phi,c} = \frac{1}{2}C_h T_{h1}^2 - \frac{1}{2}C_h T_{h2}^2 + \frac{1}{2}C_c T_{c1}^2 - \frac{1}{2}C_c T_{c2}^2 \quad (12)$$

$$R_{h,HXs} = \frac{\dot{E}_{h\phi,HXs}}{\dot{Q}_{HXs}^2} \quad (13)$$

Substituting Eq. (12) into Eq. (13) and using the relations in Eq. (11) gives:

$$R_{h,HXs} = \frac{1}{2}\frac{(T_{h1} + T_{h2}) - (T_{c1} + T_{c2})}{\dot{Q}_{HXs}} \quad (14)$$

According to energy balances in Eq. (11),

$$T_{h2} = T_{h1} - \frac{\dot{Q}_{HXs}}{C_h}, \quad T_{c2} = T_{c1} + \frac{\dot{Q}_{HXs}}{C_c} \quad (15)$$

Substitute Eq. (15) into Eq. (14) to get:

$$R_{h,HXs} = \frac{T_{h1} - T_{c1}}{\dot{Q}_{HXs}} - \frac{1}{2}\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \quad (16)$$

For fixed heat capacity rates of the hot and cold fluids, from Eq. (16), it is clear that when the inlet temperatures of the hot and cold fluids are given, the heat transfer rate $\dot{Q}_{HXs}$ is the monodrome function of the equivalent thermal resistance of heat exchanger couple and the heat transfer rate increases as the thermal resistance decreases, i.e. the smaller the equivalent thermal resistance, the larger the heat transfer rate. Similarly, for fixed heat capacity rates of the two fluids and fixed heat transfer rate, the inlet temperature difference between hot and cold fluids decreases as the thermal resistance decreases, i.e. the smaller the equivalent thermal resistance, the smaller the fluid inlet temperature difference.

3.2. Optimal heat capacity rate of the medium fluid

Assume that both heat exchangers A and B in Fig. 2 are counter-flow heat exchangers. If the following parameters of (1) cold and hot fluids inlet temperatures $T_{h1},\ T_{c1}$, (2) fluids heat capacity rates $C_h,\ C_c$ and (3) heat exchangers thermal conductance $K_hA_h,\ K_cA_c$ are given, the thermal performance of heat exchanger couple is determined by the medium fluid heat capacity rate C_m. At first glance, it seems that the bigger the medium fluid heat capacity rate C_m, the better the thermal performance of the heat exchanger couple. However, it is not the fact. Due to the energy balance of the two heat exchangers, there exists an optimum value for C_m.

The heat transfer equations for heat exchanger A on the source side and the heat exchanger B on the load side are:

$$\dot{Q}_h = K_hA_h\Delta T_{m,h} = K_hA_h\frac{(T_{h1}-T_{m1})-(T_{h2}-T_{m2})}{\ln(T_{h1}-T_{m1})-\ln(T_{h2}-T_{m2})} \tag{17}$$

$$\dot{Q}_c = K_cA_c\Delta T_{m,c} = K_cA_c\frac{(T_{m1}-T_{c2})-(T_{m2}-T_{c1})}{\ln(T_{m1}-T_{c2})-\ln(T_{m2}-T_{c1})} \tag{18}$$

Substitute Eq. (11) into Eq. (17) to get:

$$\dot{Q}_h = K_hA_h\frac{\frac{\dot{Q}_h}{C_h}-\frac{\dot{Q}_h}{C_m}}{\ln(T_{h1}-T_{m1})-\ln(T_{h2}-T_{m2})} \tag{19}$$

Eliminate $\dot{Q}_h$ on both sides of Eq. (19) and rearrange to get:

$$\ln\frac{T_{h1}-T_{m1}}{T_{h2}-T_{m2}} = K_hA_h\left(\frac{1}{C_h}-\frac{1}{C_m}\right) \tag{20}$$

Let

$$\mu_h = \frac{1}{C_h}-\frac{1}{C_m} \tag{21}$$

$$M = \exp(K_hA_h\mu_h) \tag{22}$$

with μ_h and M, Eq. (20) can be rewritten as

$$\frac{T_{h1}-T_{m1}}{T_{h2}-T_{m2}} = \exp(K_hA_h\mu_h) = M \tag{23}$$

For the heat exchanger B, let

$$\mu_c = \frac{1}{C_m}-\frac{1}{C_c} \tag{24}$$

$$N = \exp(K_cA_c\mu_c) \tag{25}$$

Following the same steps used for analyzing high-temperature heat exchanger A, it is easy to get:

$$\frac{T_{m1}-T_{c2}}{T_{m2}-T_{c1}} = \exp(K_cA_c\mu_c) = N \tag{26}$$

Combine Eq. (23) and Eq. (26),

$$T_{m1}-T_{m2} = \frac{(1-N)(T_{h1}-MT_{h2})+(1-M)(NT_{c1}-T_{c2})}{M-N} \tag{27}$$

Consider that

$$T_{m1}-T_{m2} = \frac{\dot{Q}_m}{C_m} = \frac{\dot{Q}_{HXs}}{C_m} \tag{28}$$

Substitute Eqs. (28) and (15) into Eq. (27), the heat transfer rate can be expressed as

$$\dot{Q}_{HXs} = \frac{(1-N)(1-M)(T_{h1}-T_{c1})}{\frac{M-N}{C_m}-\frac{M(1-N)}{C_h}+\frac{1-M}{C_c}} = \frac{T_{h1}-T_{c1}}{\frac{\mu_h}{M-1}+\frac{\mu_c}{N-1}+\frac{1}{C_h}} \tag{29}$$

Substitute Eq. (29) into Eq. (16), the equivalent thermal resistance of the heat exchanger couple is obtained as

$$R_{h,HXs} = \frac{\mu_h}{M-1}+\frac{\mu_c}{N-1}+\frac{1}{2}\left(\frac{1}{C_h}-\frac{1}{C_c}\right) \tag{30}$$

From Eq. (30), it is clear that the thermal resistance is independent of fluid temperatures and is a function of the heat exchanger thermal conductance and the fluid heat capacity rates. According to the minimum thermal resistance principle, when the overall equivalent thermal resistance of heat exchanger couple is minimized, the thermal performance (heat transfer rate) is maximized. Make the partial derivative of $R_{h,HXs}$ with respect to C_m and let it equal to zero to obtain the following equation:

$$\frac{\partial R_{h,HXs}}{\partial C_m} = \frac{(M-1)-M\cdot\ln M}{(M-1)^2}-\frac{(N-1)-N\cdot\ln N}{(N-1)^2} = 0 \tag{31}$$

Solving Eq. (31) leads to the optimal heat capacity rate of medium fluid $C_{m,opt}$:

$$M = N, \quad C_{m,opt} = \frac{K_hA_h+K_cA_c}{\frac{K_hA_h}{C_h}+\frac{K_cA_c}{C_c}} \tag{32}$$

Eq. (32) is the analytical expression of optimal heat capacity rate of medium fluid, which is the function of heat exchanger thermal conductance and fluid heat capacity rate. The parameter variation of heat exchanger couple can be divided into four types:

(1) $C_h = C_c, \quad K_hA_h = K_cA_c, \quad C_{m,opt} = C_h = C_c;$
(2) $C_h = C_c, \quad K_hA_h \neq K_cA_c, \quad C_{m,opt} = C_h = C_c;$
(3) $C_h \neq C_c, \quad K_hA_h = K_cA_c, \quad C_{m,opt} = \frac{2}{\frac{1}{C_h}+\frac{1}{C_c}} = \frac{2C_hC_c}{C_h+C_c};$
(4) $C_h \neq C_c, \quad K_hA_h \neq K_cA_c, \quad$ let $\beta = K_cA_c/K_hA_h,$
then $C_{m,opt} = (1+\beta)\frac{C_hC_c}{C_c+\beta C_h}.$

For the first three types, $C_{m,opt} = \frac{2}{\frac{1}{C_h}+\frac{1}{C_c}}$, which means the optimal heat capacity rate of the medium fluid (hydronic fluid) equals to the equivalent capacity rate of series-connected high temperature fluid heat capacity rate and low temperature fluid heat capacity rate, which is similar to the series connection of two capacitors. For the fourth type, the optimal value can be regarded as "thermal conductance-weighted" series-connected heat capacity rate. In the literature [7], the high temperature fluid heat capacity rate equals to that of the low temperature fluid, i.e. $C_h = C_c$, through detailed modeling and calculation, the authors conclude that the optimal

Table 1
Calculation cases for effect of medium fluid heat capacity rate on heat exchanger couple performance.

Case No.	HX A			HX B			Medium fluid
	T_{h1}/K	C_h/(kW/K)	K_hA_h/(kW/K)	T_{c1}/K	C_c/(kW/K)	K_cA_c/(kW/K)	$C_{m,opt}$/(kW/K) by Eq. (32)
A1	323	1.2	2.5	297	1.2	2.5	1.2
A2	323	1.2	3	297	1.2	2	1.2
A3	323	1.2	2.5	297	0.6	2.5	0.8
A4	323	0.6	1	297	1.2	4	1

heat capacity rate of hydronic fluid should meet $C_{m,opt} = C_h = C_c$. Their conclusion is identical with that in type (2) and is the special case of present result of Eq. (32).

In order to analyze the effects of medium fluid heat capacity rate on heat transfer performance of heat exchanger couple, for four typical cases listed in Table 1, numerical calculations based on energy conservation equation (11) and heat transfer equations (17) and (18) are carried out to obtain the variation of heat transfer rate, equivalent thermal resistance with the medium fluid heat capacity rate for the heat exchanger couple, as shown in Fig. 3.

The vertical coordinate is normalized value so that different physical quantities can be plotted in one figure. It can be found in Fig. 3 that the overall thermal resistance $R_{\mathrm{h},HXs}$ decreases at the beginning, reaches its minimum, then increases again and trends to be a constant value when the medium fluid heat capacity rate increases, while the heat transfer rate increases at first, reaches its maximum and then decreases and trends to be constant. Meanwhile, the maximum heat transfer rate corresponds to the minimum equivalent thermal resistance, which is predicted by the minimum thermal resistance principle. The medium fluid heat capacity rate optimal values from the numerical calculation agree with the results from theoretical optimal results from Eq. (32) as listed in Table 1.

It should be pointed out that the optimal medium fluid heat capacity rate and the corresponding maximum heat transfer rate can also be derived directly from the heat transfer rate expression of Eq. (29) without introducing the equivalent thermal resistance. However, the use of equivalent thermal resistance can reveal the physical meaning of the optimal heat capacity rate of medium fluid, at which the thermal resistance of the heat exchanger couple or the heat transfer irreversibility is minimized, and consequently the heat transfer rate is maximized.

3.3. Optimal allocation of thermal conductance

When such parameters as (1) fluid inlet temperatures T_{h1}, T_{c1}, (2) fluid heat capacity rates C_h, C_c, C_m and (3) the total thermal conductance $\sum KA = K_hA_h + K_cA_c$ are given, the allocation of thermal conductance between the two heat exchangers can be optimized by using the minimum thermal resistance principle to make the equivalent thermal resistance of heat exchanger couple minimized, and accordingly the heat transfer rate maximized.

Analogical to the analysis process used for optimizing the medium fluid heat capacity rate, the optimization of thermal conductance allocation can also be carried out in two ways, i.e. theoretical derivation and numerical calculation. Make the partial derivative of $R_{\mathrm{h},HXs}$ with respect to the low-temperature heat exchanger's thermal conductance K_cA_c and let it equal to zero:

$$\frac{\partial R_{\mathrm{h},HXs}}{\partial (K_cA_c)} = \frac{M\mu_h^2}{(M-1)^2} - \frac{N\mu_c^2}{(N-1)^2} = 0 \tag{33}$$

For the solution of Eq. (33), the parameters of heat exchanger couple can be divided into two classes: (1) $C_h = C_c$ and (2) $C_h \neq C_c$. Table 2 lists two calculation cases for evaluating the thermal conductance allocation of the heat exchanger couple with the total thermal conductance being kept at $\sum KA$ = 5 kW/K. The numerical results for the two cases are shown in Fig. 4. With the increase of the thermal conductance of heat exchanger B, the overall thermal resistance $R_{\mathrm{h},HXs}$ decreases at the beginning, reaches its minimum and then increases again while for the heat transfer rate $\dot{Q}_{HXs}$, it increases first, reaches its maximum and then decreases. Again, the maximum heat transfer rate corresponds to the minimum overall thermal resistance. For cases B1 and B2, the optimal fraction of thermal conductance allocations for heat exchanger B are 0.5 and 0.524, respectively, which agree with the theoretical results from Eq. (33).

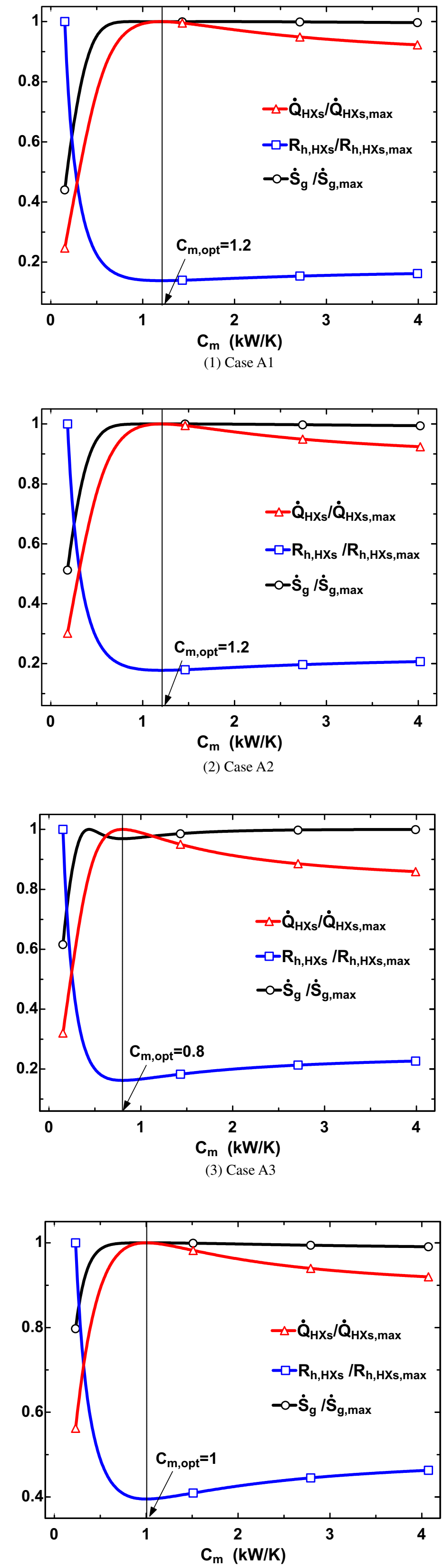

Fig. 3. Normalized heat transfer rate, equivalent thermal resistance and entropy generation rate versus the medium fluid heat capacity rate (each figure corresponds to a case in Table 1).

Table 2
Calculation cases for the effect of the thermal conductance allocation on the heat exchanger couple performance.

Case No.	HX A		HX B		Medium fluid	Allocation
	T_{h1}/K	$C_h/(\mathrm{kW/K})$	T_{c1}/K	$C_c/(\mathrm{kW/K})$	$C_m/(\mathrm{kW/K})$	ε_{opt} by Eq. (33)
B1	323	1.2	297	1.2	0.6	0.5
B2	323	1.2	297	0.6	0.4	0.524

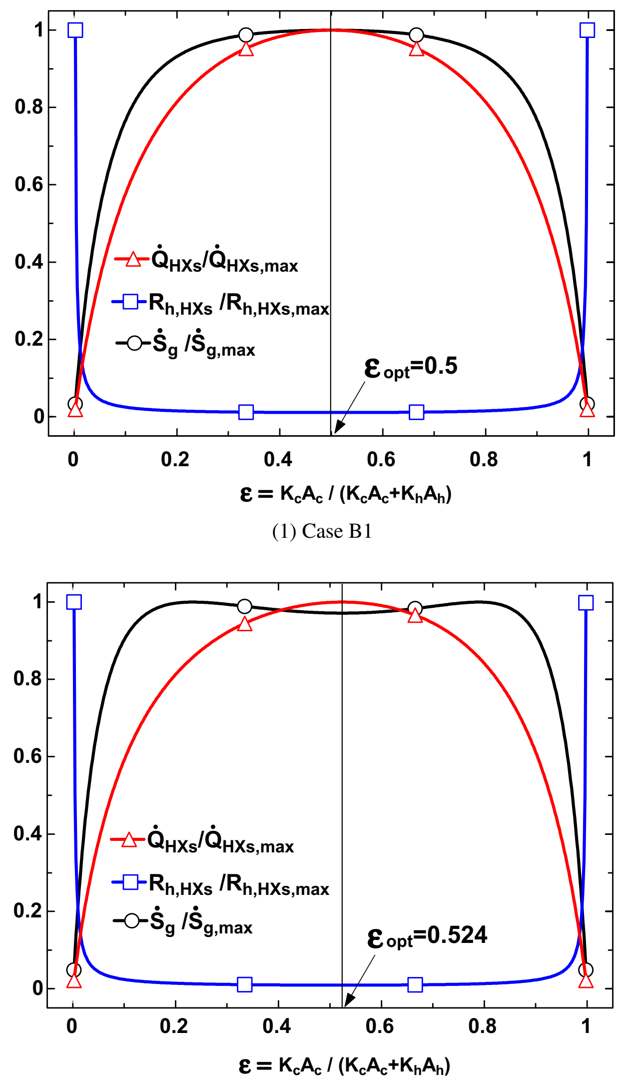

Fig. 4. Normalized heat transfer rate, equivalent thermal resistance and entropy generation rate versus the thermal conductance allocation (each figure corresponds to a case in Table 2).

4. Comparison with minimum entropy generation principle

Bejan [15–17] introduced the concept of convective heat transfer irreversibility due to finite temperature difference as well as fluid friction based on the second law of thermodynamics. The minimum entropy generation is taken as the optimization criterion for heat transfer or for heat exchangers, which is referred to as thermodynamic optimization. The entropy generation rate in a single counterflow heat exchanger is

$$\dot{S}_g = C_h(\ln T_{h2} - \ln T_{h1}) + C_c(\ln T_{c2} - \ln T_{c1}) \tag{34}$$

From then on, researchers [18–20] used the minimum entropy generation principle to optimize different types of heat exchangers. They believe that the optimal heat exchanger performance corresponds to the minimum entropy generation.

In order to discuss whether the minimum entropy generation principle holds for the thermal performance optimization of coupled heat exchangers, we need to calculate the entropy generation rate in the heat exchanger couple. Since there is no contribution to the entropy generation rate from the medium fluid in ideal condition, the expression for the entropy generation rate for a heat exchanger couple is the same as Eq. (34).

The calculated curves for the normalized entropy generation rate versus the heat capacity rate and thermal conductance allocation are also illustrated in Figs. 3 and 4, respectively. It can be seen that the minimum entropy generation rate does not correspond to the maximum heat transfer rate. This fact can be validated by the mathematic analysis of Eq. (34). Make partial derivative of $\dot{S}_g$ with respect to the medium fluid heat capacity rate C_m to get,

$$\frac{\partial \dot{S}_g}{\partial C_m} = C_h \frac{\partial \ln T_{h2}}{\partial C_m} + C_c \ln \frac{\partial \ln T_{c2}}{\partial C_m} \tag{35}$$

Substitute Eqs. (15) and (29) into Eq. (35) to get:

$$\frac{\partial \dot{S}_g}{\partial C_m} = \frac{\frac{(M-1)-M\ln M}{(M-1)^2} - \frac{(N-1)-N\ln N}{(N-1)^2}}{C_m^2\left(\frac{\mu_h}{M-1} + \frac{\mu_c}{N-1} + \frac{1}{C_h}\right)^2}\left(\frac{1}{T_{h2}} - \frac{1}{T_{c2}}\right)(T_{h1} - T_{c1}) \tag{36}$$

If $\frac{\partial \dot{S}_g}{\partial C_m} = 0$, then

$$\frac{(M-1) - M\ln M}{(M-1)^2} - \frac{(N-1) - N\ln N}{(N-1)^2} = 0 \tag{37a}$$

$$T_{h2} - T_{c2} = 0 \tag{37b}$$

These are the two equations that the extremum of the entropy generation rate should meet and Eq. (37a) is the same as Eq. (31) in the thermal resistance analysis, and Eq. (37b) can be expressed as follows:

$$T_{h1} - \frac{T_{h1} - T_{c1}}{\frac{\mu_h}{M-1} + \frac{\mu_c}{N-1} + \frac{1}{C_h}} \frac{1}{C_h} = T_{c1} + \frac{T_{h1} - T_{c1}}{\frac{\mu_h}{M-1} + \frac{\mu_c}{N-1} + \frac{1}{C_h}} \frac{1}{C_c} \tag{38}$$

Let

$$\Delta T_{out} = T_{h2} - T_{c2} = f(C_m) \tag{39}$$

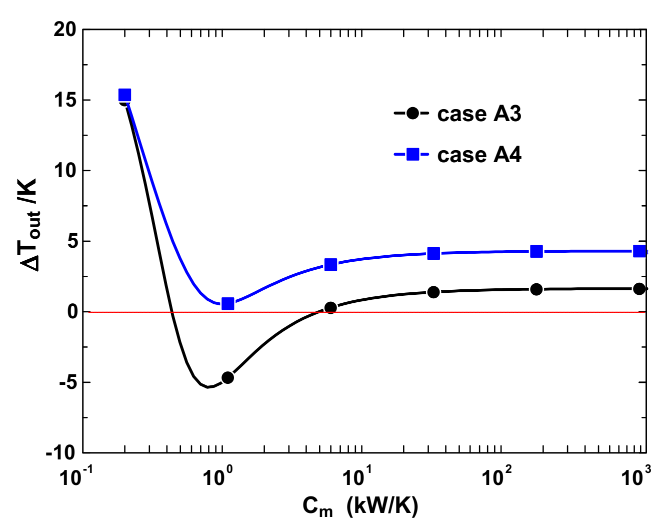

Fig. 5. Outlet temperature difference of high-temperature and low-temperature fluids versus the medium fluid heat capacity rate.

Taking Cases A3 and A4 listed in Table 1 as examples, the variations of outlet temperature difference ΔT_{out} of hot and cold fluids with the medium fluid heat capacity rate are shown in Fig. 5.

The curve of fluid outlet temperature difference for Case A3 intersects the zero line twice so there are two solutions to Eq. (37b) which corresponding to two entropy generation extrema. Together with one another extremum corresponding to the solution of Eq. (37a), there are three entropy generation extrema. The curve of fluid outlet temperature difference for Case A4 has no intersection with the zero line. Consequently there's only one entropy generation extremum corresponding to the solution of Eq. (37a) which locates at the same position as the minimum thermal resistance. However, this extremum entropy generation rate is the maximum rather than the minimum, as shown in Fig. 3 (4).

To sum up, there is no certain correspondence between the minimum entropy generation rate and the optimal thermal performance of heat exchanger couple.

5. Concluding remarks

(1) Based on the entransy and entransy dissipation concepts, the equivalent thermal resistance is defined for a heat exchanger couple, which measures the irreversibility of heat transfer for the purpose of heating or cooling, rather than heat to work conversion. Therefore, the thermal performance of heat exchanger couple can be optimized by using minimum thermal resistance principle, that is, minimizing its thermal resistance may result in the maximum heat transfer rate in the heat exchanger couple.

(2) Both theoretical analysis and numerical calculation indicate that there exists an optimal heat capacity rate of medium fluid for fixed heat exchanger thermal conductance and high/low temperature fluid capacity rates, and an optimal thermal conductance allocation for fixed medium fluid heat capacity rate and total thermal conductance of heat exchanger couple. These optimal values correspond to the minimum thermal resistance and, consequently, the maximum heat transfer rate.

(3) The reciprocal of the optimal heat capacity rate of medium fluid for the best thermal performance of the heat exchanger couple equals to the sum of the reciprocal of the individual capacity rates of hot and cold fluids, just like the case of two electrical capacitors in series.

(4) Calculations of the entropy generated in the heat exchanger couple show that there is no one-to-one correspondence of the minimum entropy generation rate and the maximum heat transfer rate. This indicates that the minimum entropy generation cannot be taken as the optimization criterion of the heat exchanger couple.

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